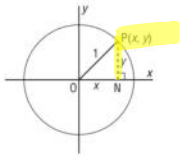
6.1 Trigonometric Ratios for Any Angle in Standard Position (cont'd)

Name:
Date:

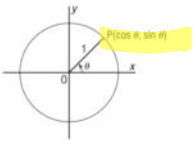
Part A: Unit Circle

Equation of the Unit Circle:



A circle, centre at origin, radius 1,
We use Pythag:
$x^2 + y^2 = 1^2$
$\therefore x^2 + y^2 = 1$

When $P(x, y)$ is the point where the terminal arm of angle θ intersects the unit circle, then:



$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$ $\tan\theta = \frac{o}{a}$

$\therefore \sin\theta = \frac{y}{1}$ $= \frac{x}{1}$ $\tan\theta = \frac{y}{x}$

$\therefore \sin\theta = y$ $\therefore \cos\theta = x$

Reciprocal Trigonometric Ratios:

The reciprocal of $\cos\theta$ is: secant θ $\therefore \sec\theta = \frac{1}{x}$

The reciprocal of $\sin\theta$ is: cosecant θ $\therefore \csc\theta = \frac{1}{y}$

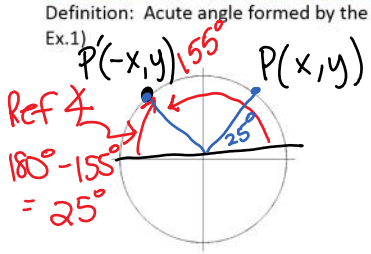
The reciprocal of $\tan\theta$ is: cotangent θ $\therefore \cot\theta = \frac{x}{y}$

Part B: Reference Angles

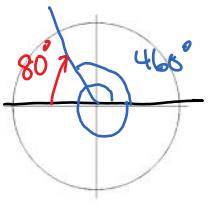
http://www.mathopenref.com/common/appletframe.html?applet=referenceangle&wid=600&ht=350

Definition: Acute angle formed by the terminal arm and x-axis.

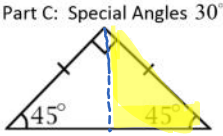
Ex.1)

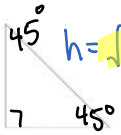


Ex.2) Reference ∠ for 460°



Part C: Special Angles $30^\circ - 45^\circ - 60^\circ$

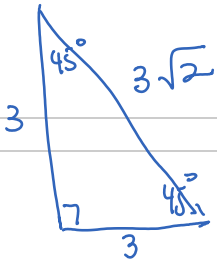


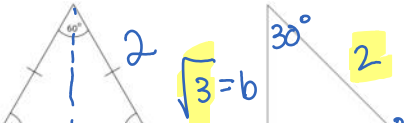


$h = \sqrt{2}$

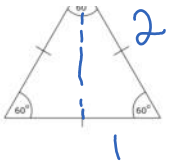
Use Pythag
$1^2 + 1^2 = h^2$
$2 = h^2$
$\therefore \sqrt{2} = h$

$\therefore 1 : 1 : \sqrt{2}$

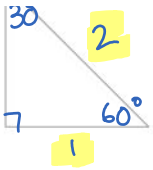




Use Pythag:
$1^2 + b^2 = 2^2$ $\therefore 1 : \sqrt{3} : 2$
$1 + b^2 = 4$



$\sqrt{3} = b$



$1^2 + b^2 = 2^2$

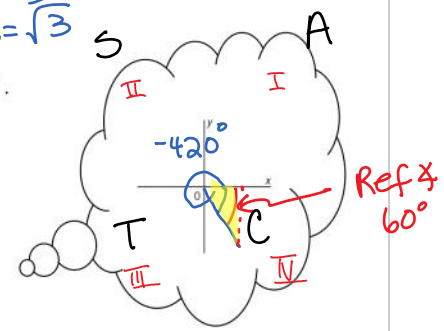
$1 + b^2 = 4$

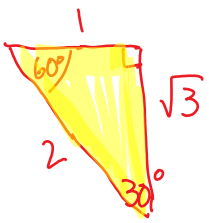
$b^2 = 3$

$\therefore b = \sqrt{3}$

Part D: Examples

Ex.3) Determine the exact values of the six trigonometric ratios for -420°.





$\sin 60^\circ = \frac{\sqrt{3}}{2} \rightarrow \sin -420^\circ = \boxed{-\frac{\sqrt{3}}{2}}$

$\cos 60^\circ = \frac{1}{2} \rightarrow \cos(-420^\circ) = \frac{1}{2}$

$\tan 60^\circ = \frac{\sqrt{3}}{1} \rightarrow \tan(-420^\circ) = -\sqrt{3}$

$\therefore \csc -420^\circ = -\frac{2}{\sqrt{3}}$ $\quad \sec(-420^\circ) = 2$

$\cot(-420^\circ) = -\frac{1}{\sqrt{3}}$

HW6.1
(p. 474)
#1-6 (all)

Ex.4) Determine the approximate values of the six trigonometric ratios for 586°, to the nearest thousandth.

$\sin 586^\circ = -0.719$

$\cos 586^\circ = -0.695$

$\tan 586^\circ = 1.036$

$\csc 586^\circ = \frac{1}{\sin 586^\circ}$

$= -1.390$

$\sec 586^\circ = \frac{1}{\cos 586^\circ}$

$= -1.440$

$\cot 586^\circ = \frac{1}{\tan 1.036}$

$= 0.966$

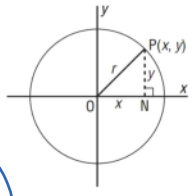
Part E: Equation of a Circle

A circle, centre the origin and radius r units, has equation: $x^2+y^2=r^2$

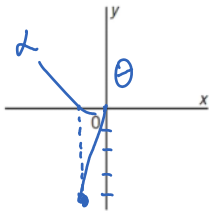
For any angle θ in standard position, with terminal point P(x,y) on a circle radius, r:

$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$ $\tan\theta = \frac{y}{x}$

$\sec\theta = \frac{r}{x}$ $\csc\theta = \frac{r}{y}$ $\cot\theta = \frac{x}{y}$



Ex.5) P(-1, -4) is a terminal point of angle θ in standard position. Determine the exact values of the six trigonometric ratios for θ.



P(-1,-4)

Let $OP = r$, $x=-1$, $y=-4$

Using Pythag:

$x^2+y^2=r^2$

$(-1)^2+(-4)^2=r^2$

$17=r^2$

$\therefore \boxed{r=\sqrt{17}}$

$\cos\theta = \frac{x}{r} = \boxed{\frac{-1}{\sqrt{17}}}$

$\sec\theta = \boxed{-\sqrt{17}}$

$\sin\theta = \frac{y}{r} = \boxed{\frac{-4}{\sqrt{17}}}$

$\csc\theta = \boxed{\frac{-\sqrt{17}}{4}}$

$\tan\theta = \frac{y}{x} = \frac{-4}{-1} = \boxed{4}$

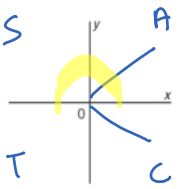
$\cot\theta = \boxed{\frac{1}{4}}$

Ex.6) Suppose $\sec\theta = 4$.

a) Determine the exact values of the other trigonometric ratios for $0^\circ \le \theta \le 180^\circ$.

$\sec\theta = 4$ Since $\cos\theta = \frac{x}{r}$

$\therefore \frac{r}{x} = \frac{4}{1}$ then $\sec\theta = \frac{r}{x}$



$\therefore x = 1,\ r = 4$, use Pythag to find y:

$x^2 + y^2 = r^2$

$1^2 + y^2 = 4^2$

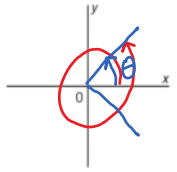
$y^2 = 15$

$\therefore y = \pm\sqrt{15}$ Since terminal arm is in Quad I $\therefore y = +\sqrt{15}$

$\cos\theta = \frac{1}{4}$ $\sin\theta = \frac{y}{r} = \frac{\sqrt{15}}{4}$ $\tan\theta = \frac{y}{x} = \frac{\sqrt{15}}{1}$

$\sec\theta = 4$ $\csc\theta = \frac{4}{\sqrt{15}}$ $\therefore \cot\theta = \frac{1}{\sqrt{15}}$

b) To the nearest degree, determine possible values of θ in the domain $-360^\circ \le \theta \le 360^\circ$.

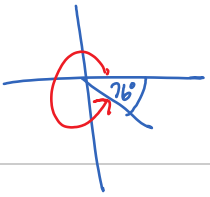


$\cos\theta = \frac{1}{4}$

$\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\theta = 76^\circ$

In Quad I:



$\therefore \theta_1 = 76^\circ$ $\theta_2 = -360^\circ + 76^\circ$

$= -284^\circ$

In Quad IV:

$\theta_3 = -76^\circ$ $\theta_4 = 360^\circ - 76^\circ$

$= 284^\circ$

$\therefore$ Possible values of θ are approx

$\pm 76^\circ, \pm 284^\circ$

HW 6.1
(p. 474) # 1-11 (all)